BOUNDARY CONDITIONS FOR DIFFUSION OF A NONEQUILIBRIUM PLASMA IN A MAGNETIC FIELD

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In the elementary theory of gases and of unmagnetized plasma [1-3], two fundamental quantities are the mean free path $\lambda = v\tau$ for the particles and the density of their random flow $J_0 = nv/4$. By means of these parameters, one can determine all transport coefficients and the corresponding macroscopic flows of particles, viscous momentum, and heat [3]. Both quantities also play an important role in the theory of wall layers (the Knudsen λ layer [4]) in which the macroscopic equations lose meaning. Matching the flows for internal and external plasma using these parameters yields virtual boundary conditions at the walls for the equations specified, the solution of which outside the λ layer agrees with the true solution. In particular, boundary conditions for simple diffusion of particles at an absorbing wall are roughly determined [5-7] from the equality of the diffusion flow J_D and the random flow J_0 of the particles in the boundary layer,

$$n_b = 2n_w = -2\lambda_D \nabla n, \ \lambda_D = (2/3)\lambda, \tag{1}$$

where n_w is the particle density at the wall.

A demonstration of the satisfactory accuracy of Eq. (1) by direct solution of the Boltzmann transport equation in all transition regions is presented in [7].

When a magnetic field was present, the expressions for the macroscopic flows were usually derived by formal means [3, 8] with the concepts of transverse mean free path $\lambda_\perp$ and random flow $J_0^\perp$ needed for derivation of the boundary conditions not yet established. It was only known [8, 9], that the Larmor radius $r_\perp = v_\perp/|\omega|$ evolves into $\lambda_\perp$ in the case of strong magnetization ($\beta = |\omega|\tau \gg 1$, where $\omega = eB/mc$ is the cyclotron frequency).

The lack of clarity in this problem led to a situation where the correct boundary conditions for a plasma in a magnetic field were even qualitatively unknown although repeated attempts were made in that direction [10-12]. The dependence of the diffusion coefficient on the field B was taken into account [10] in a boundary condition of the type (1) taken from [5, 6] while the dependence of the flow $J_0^\perp$ on B was omitted. A similar condition was also used in a recent paper [11]. The use of the flow J_0 in place of $J_0^\perp$ in that paper was clearly invalid. An attempt was made [12] to take into account the effect of the field B on the flow $J_0^\perp$ also by using the method of free paths [3]. As a result, the leading term acquired the form

$$J_0^\perp = J_0/(1+\beta^2).$$

However, this result was erroneous because of an incorrect choice of the limits of integration over the velocities c_z' and the preceding time t.

In order to determine the structural form of the parameters $\lambda_\perp$ and $J_0^\perp$ in a magnetic field, we use an analogy to the behavior of plasma in both cases of diffusion and investigate the physical significance and region of application of the newly introduced concepts. As is well known [1-3, 7], diffusion flow in a plasma without a magnetic field is equal to the difference of the random flows J_0 arriving over a diffusion mean free path,

$$J_D = J_0(-\lambda_D) - J_0(+\lambda_D) = -D\nabla n, \tag{2}$$

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where the diffusion coefficient is

$$D = \lambda v/3 = \lambda^2/3\tau. \tag{3}$$

With a magnetic field present, the transverse diffusion coefficient [3, 9] is

$$D_\perp = D/(1+\beta^2). \tag{4}$$

If we now represent Eq. (4) in the form $D_\perp = \lambda_\perp^2/3\tau$ by analogy with Eq. (3), we then find for the desired quantity $\lambda_\perp$

$$\lambda_\perp = \lambda/\sqrt{1+\beta^2}.$$

When $\beta \ll 1$, $\lambda_\perp$ transforms into λ, and we have $\lambda_\perp = r_\perp$ when $\beta \gg 1$.

We determine what the random flows $J_0^\perp$ must be in order that their difference over a path length $\pm(2/3)\lambda_\perp$, in analogy with Eq. (2), might give the correct value of the transverse diffusion flow. We find as a result

$$J_0^\perp = J_0/\sqrt{1+\beta^2}. \tag{5}$$

As should be expected, the flow $J_0^\perp$ agrees with the random flow J_0 for the particles when $\beta \ll 1$. When $\beta \gg 1$, we have $J_0^\perp = J_0/\beta$. We shall show that in the latter case the flow $J_0^\perp$ is the random flow of guiding centers.

The random flow of particles in a plasma for any β is always J_0. When $\beta \gg 1$, however, only those particles contribute to diffusion which experience collisions and, consequently, displacement of guiding centers. In deriving the flows of such particles, it is convenient to transform to a description of diffusion in the coordinate space of the guiding centers, in which cyclic flow of the particles is automatically eliminated. In this case, transverse flow of guiding centers of particles of type α in particles of type β has the form [9]

$$J_{\alpha\beta} = n_\alpha \langle \Delta X_{\alpha\beta}\rangle - \frac{1}{2}\frac{d}{dx}\left[n_\alpha \langle \Delta X_{\alpha\beta}^2\rangle\right], \tag{6}$$

in which the moments $\langle \Delta X_{\alpha\beta}^k \rangle$ are

$$\langle \Delta X_{\alpha\beta}^k\rangle = \frac{1}{\omega_\alpha^k}\int f_\alpha d\mathbf{v}_\alpha \int n_\beta f_\beta d\mathbf{v}_\beta \int_\Omega \Delta v_{\alpha y}^k u \sigma_{\alpha\beta}(u,\vartheta)\, d\Omega, \tag{7}$$

where $u = |\mathbf{v}_\alpha - \mathbf{v}_\beta|$ is the relative velocity and $\sigma_{\alpha\beta}$ is the differential scattering cross section.

Both terms in Eq. (6) go to zero in the absence of an external force and density gradient. In this case, the random jumps of guiding centers in the one direction, $\Delta X_{\alpha\beta} = \Delta v_{\alpha y}/\omega_\alpha$, where $\Delta v_{\alpha y}$ is the change in the y component of the velocity during a collision, are precisely compensated by jumps in the opposite direction. We determine this random flow of the guiding centers by considering displacements of only one sign in Eq. (7). Since the integral over collisions in the moment $\langle \Delta X_{\alpha\beta}\rangle$ is

$$\int_\Omega \Delta v_{\alpha y} u \sigma_{\alpha\beta}(u,\vartheta)\, d\Omega = -\frac{\mu}{m_\alpha} u_y u \sigma_1(u),$$

where μ is the reduced mass and σ_1 is the cross section for momentum transfer, we note from Eq. (7) that it is sufficient to perform averaging in $\langle \Delta X_{\alpha\beta}\rangle$ over the hemisphere $u_y > 0$ in **u** space in order to determine the specified flow. Transition in $\langle \Delta X_{\alpha\beta}\rangle$ to the center-of-inertia variables **u** and $\mathbf{V}_0$ [9] and integration over $\mathbf{V}_0$ in the half-space **u** yields the expression,

$$J_{\alpha 0}^\perp = J_\alpha^0 \frac{\nu_{\alpha\beta}}{|\omega_\alpha|}, \tag{8}$$

for the random flow of the guiding centers of the particles α in which the averaged collision frequency has the form

$$\nu_{\alpha\beta} = \frac{8}{3\sqrt{\pi}}\frac{\mu n_\beta}{m_\alpha}\left(\frac{\mu}{2T_{\alpha\beta}}\right)^{5/2}\int_0^\infty u^5 \sigma_1(u)\exp\left(-\frac{\mu u^2}{2T_{\alpha\beta}}\right) du,$$

where $T_{\alpha\beta} = (m_\alpha T_\beta + m_\beta T_\alpha)/(m_\alpha + m_\beta)$ is the reduced temperature.

Comparing Eqs. (8) and (5), we confirm that for $\beta_\alpha \gg 1$, the flow $J_{\alpha o}^{\perp}$ is indeed the random flow of the guiding centers.

The generalized parameters $\lambda_\perp$ and $J_o^\perp$ introduced above are internal parameters and drop out of a macroscopic description. However, they must be taken into account in all cases where a discrete structure appears in the medium. Thus a unified approach to the analysis of phenomena in a plasma and on its boundaries is ensured for arbitrary β.

For example, a layer with a thickness $\lambda_\perp$ is a generalization to the case of a Knudsen layer in a magnetic field. In this case, the boundary condition for density of charged particles during diffusion at an absorbing wall is automatically obtained through replacement of λ, J_o, and J_D by $\lambda^\perp$, $J_o^\perp$, and $J_D^\perp$, from which we immediately find

$$n_b = 2n_w = -(4/3)\lambda_\perp \nabla_\perp n. \tag{9}$$

Equation (9) is completely analogous to Eq. (1) and, in contrast to [10-12], yields a qualitatively correct structural dependence on the magnetization parameter β.

Analogous generalization of boundary conditions can also be obtained with other equations for a nonequilibrium plasma in a magnetic field needed for systematic numerical description of high-current electrical discharges [13] and also of MHD accelerators and plasma generators.

Another important field for the application of the parameters $\lambda_\perp$ and $J_0^\perp$ is the derivation and confirmation of the applicability of macroscopic equations. Thus the condition for the applicability of the latter — the smallness of the gradients in the transverse direction in the absence of a "mixing" effect — need not be described separately for small and large β as in [8], but in the more compact form

$$\lambda_\perp \ll L_\perp. \tag{10}$$

When the field B increases, the condition (10) automatically transforms into the condition for drift description of charged-particle motion [14].

In conclusion, we point out that with the help of the parameters $\lambda_\perp$, $J_o^\perp$, and some of their generalizations, compact structural expressions have also been successfully obtained for other flows and their corresponding transport coefficients in an arbitrary magnetic field including the viscous terms, which ordinarily do not have so explicit a form. These expressions can then be refined by means of correction factors of the order of one which follow from an exact transport solution for the specific interaction law obeyed by the particles.

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INVESTIGATION OF THE CHARACTERISTICS OF THE IONIZATION CHAMBER AND THE PROPERTIES OF THE FLOW OF A GAS-DISCHARGE ION SOURCE

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A gas-discharge ion source with volumetric ionization [1] is an effective device for creating flows of a rarefied plasma, which, with high values of the specific impulse, can be used as reactive jets [2], and which, with low velocities of the flow, find application in experiments on ionospheric aerodynamics, carried out with the aim of modeling the interaction between an aircraft and the ionosphere [3,4]. When such a source is investigated as a device for creating a reactive jet, along with metals (cesium, mercury), gases are used as its working substance: xenon, argon, and nitrogen, which (as well as helium) are used as the working substance of a gas-discharge source and in experiments on ionospheric aerodynamics. Therefore, the study of the characteristics of such a source working on gases is of considerable interest.

The aim of the present work was an investigation of the characteristics of a gas-discharge ion source and the flow of plasma set up by it. High values of the principal parameters of the source (the ionic current, the coefficient of the use of the working substance, expenditures for the production of one ion, i.e., the cost of the ions, the energy efficiency) can be obtained only with the realization of determined conditions of the burning of the discharge in the ionization chamber. Here it is important to know the distribution of the potential of the plasma, determining the losses of ions in the chamber, and the form of the distribution function of the electrons, on which the efficiency of the ionization of neutral atoms depends. In experiments on ionospheric aerodynamics, the ion source must set up a flow of synthesized plasma with relatively low (~100 eV) energies of the directed motion of the ions with the required concentration in the working part of the flow. The characteristics of such flows in the absence of a magnetic field have been discussed in [3, 4]. However, for problems of ionospheric aerodynamics, there is also required the study of the parameters of the flows of a synthesized plasma propagating in magnetic fields with an intensity up to several hundred oersteds.

The investigations were made with a source with a diameter of 10 cm, a schematic diagram of which is shown in Fig. 1 (1 is the gas inlet; 2 is the cathode; 3 is the anode; 4 is the shielding grid; 5 is the accelerating grid; 6 is the ion beam; 7 is the neutralizer; 8 is the electromagnet; and 9 is the shield). The process of the efficient conversion of a neutral gas to a plasma, whose ionic component subsequently obtains the required velocity in the ionic optical system, takes place in an ionization chamber with a directly heated electron emitter. An electromagnet is used to create an intrinsic magnetic field in the ionization chamber of the source. The current I_s through its coil, equal to 1 A, corresponded to the intensity of the magnetic field at the axis of the chamber, equal to 15 Oe.

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